CLAIMS

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- 1. A method of performing an inversion operation in a cryptographic calculation with at least two auxiliary variables, the method comprising shifting (S2) a variable, then effecting a reduction (S3) by subtracting that variable from a larger variable.
 - 2. A method according to Claim 1 wherein the variables are of the same degree.
- 10 3. A method according to Claim 1 or 2 comprising updating a plurality of additional variables such that the invariances remain valid.
 - 4. A method according to any preceding claim comprising four auxiliary variables being U, V, R and S, having the invariances:

$$|S.V-R.U| = N$$

 $S.Y = U \mod N$

 $R.Y = V \mod N$.

- 5. A method according to Claim 4 comprising decreasing U and V in size, step by step until U = 1.
 - 6. A method according to Claim 5 comprising effecting the operation R.Y = 1 mod N or R = Y^{-1} mod N, as appropriate.
- 7. A method according to any preceding claim comprising operating with the Most Significant Words of the variables.
 - 8. A method according to any preceding claim comprising providing inversion (S1-S4) over GF(p).

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- 9. A method according to any preceding claim comprising providing inversion (S10-S12) over GF(2ⁿ).
- 10. A method according to any preceding claim comprising providing
 a method for long-integer division operations.
 - 11. A computer program product directly loadable into the internal memory of a digital computer, comprising software code portions for performing the method of any one or more of Claims 1 to 10 when said product is run on a computer.

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- 12. A computer program directly loadable into the international memory of a digital computer, comprising software code portions for performing the method of any one of Claims 1 to 10 when said program is run on a computer.
- 13. A carrier, which may comprise electronic signals, for a computer program of Claim 12.
- or a computer program of Claim 12 or a carrier of Claim 13.
 - 15. Apparatus for performing an inversion operation in a cryptographic calculation with at least two auxiliary variables, the apparatus comprising means to shift a variable (V, R) and means (10-17) to effect a reduction by subtraction or addition of that variable from a larger variable.
 - 16. Apparatus according to Claim 15 wherein the variables (V, R) are of the same degree without shifting.
 - 17. Apparatus according to Claim 15 or 16 comprising means to update a plurality of additional variables such that the invariance remains valid.

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18. Apparatus according to any of Claims 15 to 17 comprising means (10-13) to operate four auxiliary variables being U, V, R and S, having the invariances:

|S.V-R.U| = N

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 $S.Y = U \mod N$

 $R.Y = V \mod N.$

- 19. Apparatus according to Claim 18 comprising means (10, 11) to decrease U and V in size, step by step until U = 1.
 - 20. Apparatus according to Claim 19 comprising means (10-16) to effect the operation R.Y = 1 mod N or R = Y^{-1} mod N, as appropriate.
- 15 21. Apparatus according to any of Claims 15 to 20 comprising means to operate with the Most Significant Words of the variables.
 - 22. Apparatus for performing an inversion operation in a cryptographic calculation substantially as hereinbefore described with reference to, and/or as illustrated in, any one or more of the Figures of the accompanying drawings.
 - 23. A method of performing an inversion operation in a cryptographic calculation substantially as hereinbefore described with reference to, and/or as illustrated in, any one or more of the Figures of the accompanying drawings.